IB MYP YEAR 5

ASSESSMENT TASK

A Special Matrix

Subject:	Y10 *Extended* Mathematics	**Name : (Class)** Niron Poon	Y10Peace ( )
Topic:	**Matrices**		
Date of assessment:	**Thursday 12th January 2012**		

- **This task assesses Criteria B and D**
- **Time allowed – *one hour 40 minutes***
- **Write your answers on the lined paper/graph paper provided. GDCs are allowed.**

ADVICE:

Read the criteria descriptors and task-specific clarifications carefully before you start your work. This will give you a clear understanding of what is required and what a high quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest levels in this task.

Criterion B (in Part 1)

Levels	Task-Specific Rubric	Official IB Descriptors
0	The student does not reach a standard described by any of the descriptors given below.	
1-2	The student performs appropriate calculations (in (a) to (c), in (g), to (i), and in (k) to (m)) in order to recognize simple patterns.	The student **applies, with some guidance,** mathematical problem-solving techniques to recognize simple patterns.
3-4	The student correctly solves (d) and (e) and suggests general rules in parts (f) and (n).	The student **applies** mathematical problem-solving techniques to recognize patterns, **and suggests** relationships or general rules.
5-6	The student describes relationships (in (j) and (n)) mathematically, and connects the various relationships. If a student has drawn conclusions consistent with their findings in part 2 (g), credit ***may*** be given here.	The student **selects and applies** mathematical problem-solving techniques to recognize patterns, **describes** them as relationships or general rules, and **draws conclusions** consistent with findings.
7-8	The student is able to correctly prove mathematically all the relationships and rules seen in the task, and is successful in (j) and (n)	The student **selects and applies** mathematical problem-solving techniques to recognize patterns, **describes** them as relationships or general rules, draws the **correct conclusions** consistent with findings, and **provides justifications or a proof.**

Not Sure, Maybe 4-5

Criterion D (in Part 2)

Levels	Task-Specific Rubric	Official IB Descriptors
0	The student does not reach a standard described by any of the descriptors given below.	
1-2	The student's answers to questions (b) and (c) describe the importance of his/her findings in real life.	The student **attempts** to explain whether his/her results make sense in the context of the problem. The student **attempts to describe** the importance of his or her findings in connection to real life where appropriate.
3-4	The student's answers to questions (d) and (e) describe the importance of his/her findings in real life and comment on how they make sense in the context of the problem.	The student **correctly but briefly explains** whether his/her results make sense in the context of the problem. The student **describes the importance of** his/her findings in connection to real life where appropriate. The student **attempts to justify** the degree of accuracy of his/her results where appropriate.
5-6	The student is able to recognize and explain the applications seen in the matrix operations. In question (f), the student has explained whether the result makes sense in the context of the problem. In question (g), the student describes multiple valid methods and chooses reasonably between them.	The student **critically explains** whether his or her results make sense in the context of the problem. The student provides a **detailed explanation** of the importance of his/her findings in connection to real life where appropriate. The student **justifies** the degree of accuracy of his/her results where appropriate. The student suggests improvements to his/her method if necessary.

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A Special Matrix

Part 1

This section is assessed against criterion B only
You should spend 50-55 minutes on this section, and you are advised to make full use of your GDC

Consider the following matrices:

$L = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, $M = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ and $N = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

(a) Find the matrix L^2
(b) Find the matrix M^2
(c) Find the matrix N^2

A student, Xavi, puts forward the following hypothesis:

If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ then A^2 is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

(d) Find A^2
(e) Is Xavi right?
(f) If your answer to **(e)** is "yes", is Xavi ***always*** right? Explain
If your answer to **(e)** is "no", why is he wrong? Explain

(g) Find the matrix LM
(h) Find the matrix MN
(i) Find the matrix LN

Another student, Messi, puts forward the following hypothesis:

If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ and B is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

then the matrix AB is always of the form $\begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

(j) Prove (or disprove) Messi's hypothesis

Now go back to the original matrices.

(k) Find L^3
(l) Find L^4
(m) Find L^5
(n) What do you notice about the form of the answers to (k), (l) and (m)? Try to use the various answers you have in order to generalize.

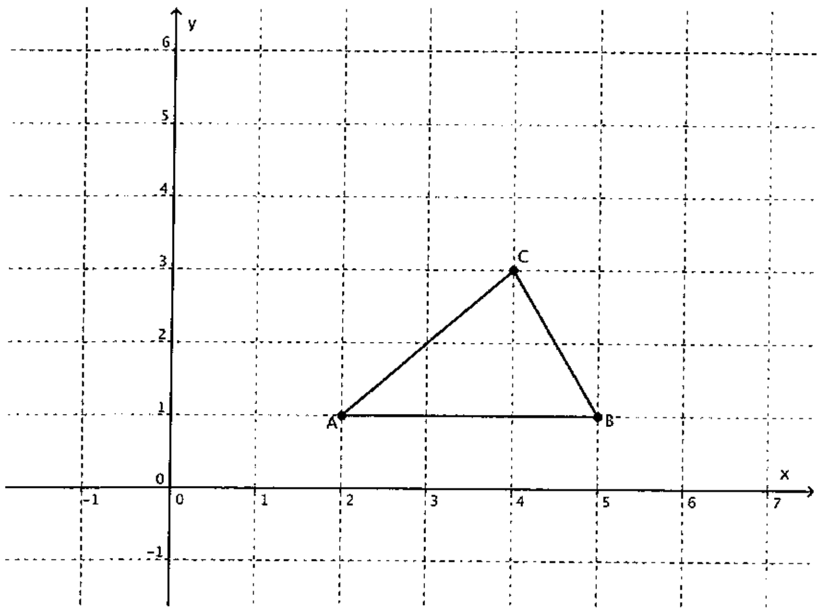
Part 2

This section is assessed against criterion D only. You should spend 40-45 minutes on this section.

Let's now look at a special case of the matrix $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$

When a = -1, the matrix is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Let's call this matrix M.

Now look at the triangle ABC below:



Write the coordinates of the triangle ABC as a 2 x 3 matrix. Call this matrix T.

(a) Calculate the matrix MT

(b) When you calculate MT, what are you really finding? In other words, what does MT represent?

(c) With the help of the graph paper provided, find the transformation described by matrix M.

(d) Calculate M^2T and comment on your answer

(e) Calculate M^3T and comment on your answer

(f) Calculate M^{-1}. Explain how this answer makes sense in the context of this problem.

(g) You are asked to calculate the matrix $M^{89998}T$
Offer at least two ways of doing this. Compare the methods and choose the most appropriate. Explain your reasoning.

Nixon Poon
Y10 Peace

A special Matrix.

Part 1

(a) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 4+1 & 2+2 \\ 2+2 & 1+4 \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 9+4 & 6+6 \\ 6+6 & 4+9 \end{pmatrix} = \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 16+9 & 12+12 \\ 12+12 & 9+16 \end{pmatrix} = \begin{pmatrix} 25 & 24 \\ 25 & 24 \end{pmatrix}$

(d) $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}^2 = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$

$$\begin{pmatrix} (a+1)(a+1)+a^2 & (a+1)(a+1)+a^2+a \\ (a+1)(a+1)+a^2+a & (a+1)(a+1)+a^2 \end{pmatrix} =$$

$= \begin{pmatrix} a^2+(a^2+1) & a^2+a^2+2a \\ a^2+a+a^2+a & a^2+(a^2+1) \end{pmatrix} = \begin{pmatrix} a^2+1a+1^2+a^2 & a^2+a+a^2+1a+1^2 \\ a^2+a+a^2+1a+1^2 & a^2+1a+1^2+a^2 \end{pmatrix}$

(e) Yes, she is always right for the form, let make it this way.

(f) Let a be 1 (There is also proofs from K.)

A

$\begin{pmatrix} 1+1 & 1 \\ 1 & 1+1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

b = other number, depending on what number a is.

A^2

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 4+1 & 2+2 \\ 2+2 & 1+4 \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} b+1 & b \\ b & 1+b \end{pmatrix}$

Look at (a)(b)(c), they all got the same personalities,

(b) $\begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$ (c) $\begin{pmatrix} 25 & 24 \\ 24 & 25 \end{pmatrix}$

$\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$ $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

b = aa

(g) $LM = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 6+2 & 4+3 \\ 3+4 & 2+6 \end{pmatrix} = \begin{pmatrix} 8 & 7 \\ 7 & 8 \end{pmatrix}$

(h) $MN = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 12+6 & 9+8 \\ 8+9 & 6+12 \end{pmatrix} = \begin{pmatrix} 18 & 17 \\ 17 & 18 \end{pmatrix}$

(i) $LN = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 8+3 & 6+4 \\ 4+6 & 3+8 \end{pmatrix} = \begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix}$

(j) (g) (h) (i) might already have proven that, but for instand, lets check it with unknowns.

$$\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix} \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix} = \begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$$

$C = ab+b+ab+a$

$$= \begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix} = \begin{pmatrix} ab+b+a \; ab+1^2 & ab+b+ab+a \\ ab+b+ab+a & \underbrace{ab+b+a+ab}_{c}+1^2 \end{pmatrix} =$$

$= c+1$

Applying it to (i)

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix} = \begin{pmatrix} 10+1 & 10 \\ 10 & 1+10 \end{pmatrix} = \begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$$

Z to the same

$$\begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 74 & 73 \\ 73 & 74 \end{pmatrix}$$

They called it c, but actually it $(a \times b)$

Proof equations: $(a+1)\ (a) \quad (b+1)\ (b)$

$(ab+1^2) = (ab+1) \longrightarrow \begin{pmatrix} ab+1 & ab \\ ab & ab+1 \end{pmatrix}$

$a+1$

$(a \times b) = (ab)$

$= \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$

a

Nixon Poon
Y10 Peace.

(k) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^2 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

$\begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 10+4 & 5+8 \\ 8+5 & 4+10 \end{pmatrix} = \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$

(l) $\begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 28+13 & 14+26 \\ 26+14 & 13+28 \end{pmatrix} = \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$

Pattern found: in $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ case, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} d+a+b & d+b+a \\ c+b+a & d+a+b \end{pmatrix}$

(m) use pattern $\begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$

(k) is the same way, because it more about 2×1, and I expected to see if I was right in (m)

Park 2

$\begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 82+40 & 41+80 \\ 80+41 & 40+82 \end{pmatrix} = \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$

I was right then.

(a) Matrix T $= \begin{pmatrix} 2 & 4 & 5 \\ 1 & 3 & 1 \end{pmatrix}$

$\begin{pmatrix} 2 & 4 & 5 \\ 1 & 3 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -2 & -4 & -5 \\ -1 & -3 & -1 \end{pmatrix}$ ← change of x and y value, negative makes big changes

(c)+(b) Negative, a xy → -xy opposition movement, see in graph.

(d) M^2T $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 & 5 \\ 1 & 3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 2 & 4 & 5 \\ 1 & 3 & 1 \end{pmatrix}$

(e) $M^3T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

$= \begin{pmatrix} -2 & -4 & -5 \\ -1 & -3 & -1 \end{pmatrix}$

(f) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(g) First way : M times 89998 times

Second way : Pattern

$M^2 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

M^3 goes back to $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

M^4 is the same as M^2, (we found that even number is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

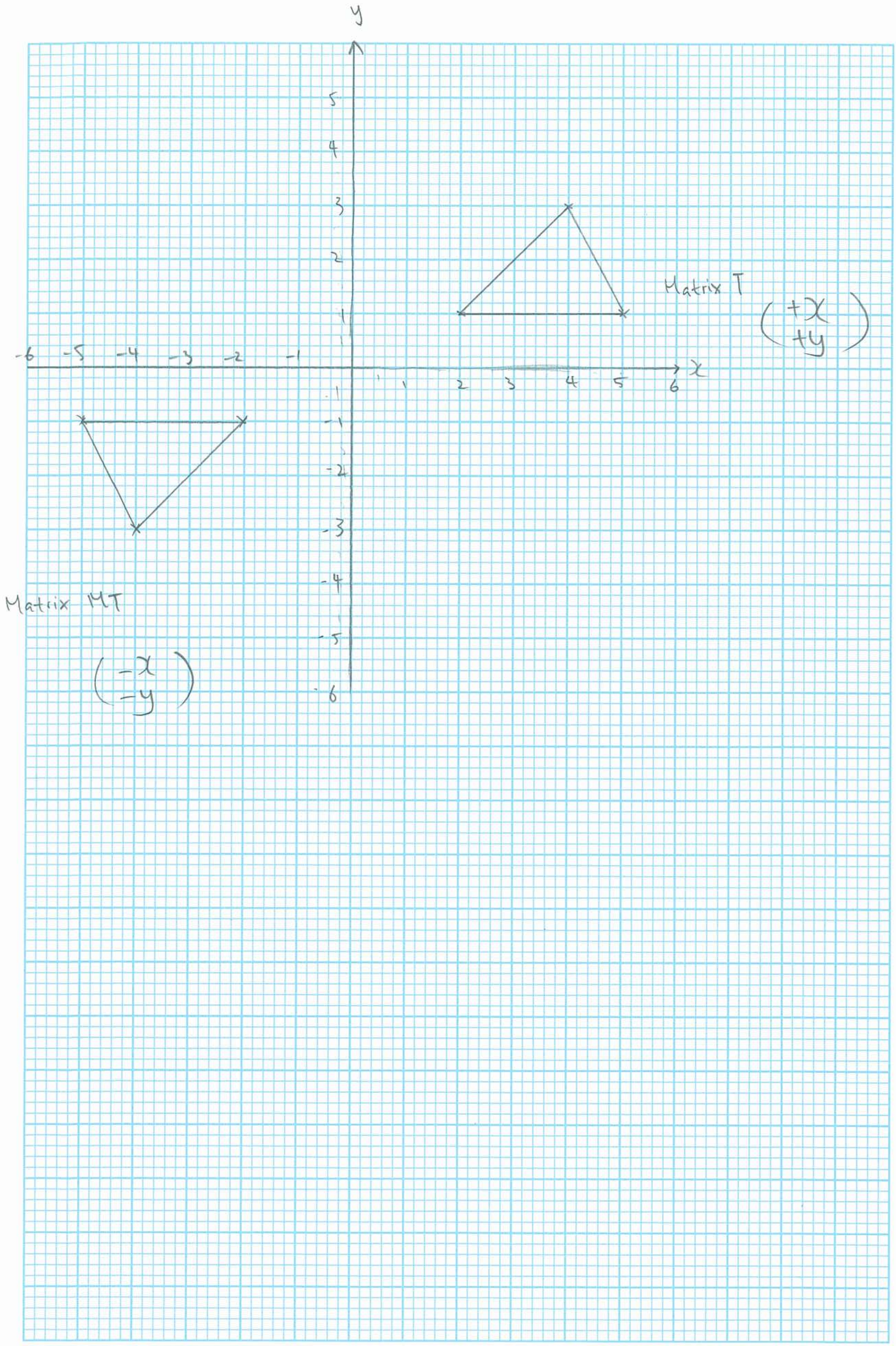
that means $M^{(\text{even number})}$ will always get $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ as we see its just repeating.

M^{89998} is an even number, so its probably $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Second way is definetely better, because it faster, and easier to understand. First way will waste a lot of time, and it has the result that the (second way) has

Part 2

(b)


y
x
Matrix T
$\begin{pmatrix} +x \\ +y \end{pmatrix}$
Matrix MT
$\begin{pmatrix} -x \\ -y \end{pmatrix}$